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working paper
department
of economics

THE STABILITY OF GENERAL EQUILIBRIUM:
RESULTS AND PROBLEMS

By

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Number 153

March 1975

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1. Introduction: The Importance of the Problem

This is a lecture about an important subject which is not in good shape -- the study of the stability of general equilibrium. In it, I shall give a partial survey of the important developments in the history of the subject and then go on to discuss my own recent work. Unfortunately, my opening characterization of the state of the art applies to that work at least as much as to anything else and much of my time will be spent discussing the all too evident problems which remain to be solved.

There was a time, about fifteen years ago or so, when the stability of general equilibrium was a hot topic among mathematical economists. Spurred on by important results of Arrow and Hurwicz, in particular,[2] there was a flurry of papers, some of them of lasting value, some of them, it now seems,

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[1] Paper presented as the Frank W. Paish Lecture at the meetings of the Association of University Teachers of Economics, Sheffield, U.K., April 1975. This paper was supported by National Science Foundation Grant GS 43185 to the Massachusetts Institute of Technology. Research on this paper was financed by the National Bureau of Economic Research, which is not, however, responsible for any of the statements made or views expressed herein. It forms part of an ongoing project which the Bureau hopes to publish when complete, pending the approval of its board of directors.

[2] Arrow and Hurwicz [4], and Arrow, Block and Hurwicz [3]. I discuss this work below. An excellent survey of work up to 1962 is given in Negishi [27]. Arrow and Hahn [2] has a later discussion, also excellent.

not very interesting minor extensions of rather restricted results. Nowadays, however, the subject, if not actually disreputable, is at least not very fashionable. Since, as I have indicated, the subject is not in good shape, it may well be that this fall from favor is due to the considerable difficulty with which even small useful results are to be obtained; others may be more adept than I at making marginal maximizing calculations in the allocation of effort. Nevertheless, I am convinced that the problems involved in stability analysis are of central importance to economic theorists, and, indeed, to economists generally. The fact that we know so little about the answers should not blind us to the importance of the questions. This should be kept in mind throughout my talk which will mostly concern the inadequacies of our knowledge. It seems appropriate, therefore, that I spend a few minutes at the outset in reminding you why the subject is an important one.

There are, I think, three related reasons for such importance (all, of course, closely derived from reasons which make general equilibrium analysis as a whole an important subject). In the first place, consider what might be called the central policy prescriptions of microeconomics. Much of what economists have to say about the results of competition, the usefulness or lack thereof of government intervention, and the role of the price system is based on propositions about general equilibrium. These are chiefly what have in modern times been rigorously formulated as the central propositions of welfare economics concerning the relations between Pareto-optima on the one hand and competitive equilibria on the other. Usually implicitly, to be sure, the propositions which may be the single most important set of ideas which economists have to convey to laymen are

based on propositions about the optimality of competitive equilibrium and, therefore, on its attainability, for there is something odd about advice which is implicitly based on how good or efficient some equilibrium point would be if there is no way to get there from here.

This sort of implicit reliance on stability propositions, however, is not confined to microeconomic policy prescriptions. Important issues of macroeconomics are also, at bottom, general equilibrium issues and therefore also involve stability questions. Let me give just two examples.

The central question which concerned Keynes in the General Theory was that of how there could be a stable equilibrium with less than full employment. That is preeminently a question of general equilibrium analysis and of general equilibrium stability analysis, in particular. The fact that Keynes and later macro-economists (undoubtedly quite wisely) did not use general equilibrium tools to answer it ought not to obscure the fact that a rigorous answer must surely be in general equilibrium terms. I yield to few economists in my ignorance of the causes of or cures for the situation of combined recession and inflation which has troubled our economies in recent years, but I cannot help suspecting that the apparent partial failure of macroeconomic analysis to cope with it may stem from the attempt to go on dealing at an aggregate level with general equilibrium questions involving detailed relatively micro-considerations.

To take a different example, consider economic issues surrounding the question of Britain's continued membership in the EEC. The alternative fates of the British economy depending on staying in or pulling out are obviously questions of displacement of general equilibrium and hence of stability, even if we are hardly able to analyze them as such.

The third reason that that study of the stability of general equilibrium is important is closely related to the first one and, nowadays, probably applies to stability analysis more than to other, better understood branches of general equilibrium analysis. An extremely prominent economist long ago remarked to me in passing that the study of stability is unimportant because it is obvious that the economy is stable and, if it isn't, we are all wasting our time. I pass over the question of whether it really is obvious that the economy is stable and observe that the issue of time-wasting by economists is not one of whether the economy is stable but rather of whether the theory is. A principal reason for studying general equilibrium in the first place is to examine the consistency of partial equilibrium analyses. Having powerful theories of the firm, the household, and the market, may not be very useful if all those theories cannot be true at the same time.

Clearly, the heart of this important consistency question lies in the existence of general equilibrium, and existence theory, fortunately, is a subject which is in pretty satisfactory shape. Nevertheless, there is a sense in which the consistency question cannot be regarded as settled without a satisfactory analysis of stability. It is no use knowing that there exist points at which all partial equilibrium propositions can be jointly true, if such points are not attainable. Hence the question of the stability of general competitive equilibrium is a vital one for economic theorists, particularly if the economy is stable, but not only then. If general equilibrium turns out to be stable only under a very restrictive set of assumptions, then, indeed, we will all have been wasting our time, for there will be something wrong with the partial theory that we think we understand.

Hence, for a number of reasons, the stability of general equilibrium is a question of great importance. As I go on to consider what we know about it and as you are tempted to grow impatient with the sorry state of that knowledge, please bear in mind that every economist continually behaves as though the unsolved questions I am addressing had long ago been satisfactorily answered.

2. Tâtonnement: Local Stability

In my view, there have been four really important developments in the history of modern stability analysis. These are: the realization that the subject was one which had to be studied in a context with a formal dynamic structure; the realization that global, rather than simply local results could be obtained; the introduction of non-tâtonnement processes; and, closely related to this, the insight that attention paid to specifying the disequilibrium processes involved could lead to far more satisfactory results than could be obtained by restricting the excess demand functions. In some ways, the analyses resulting from each of these steps have made increasing use of the economic underpinnings of the stability problem (largely, but not exclusively, Walras' Law) and have led to correspondingly more and more satisfactory results.

The history of modern stability analysis begins with Samuelson [29] and [30] who, in considering what Hicks [21] had called "perfect stability" observed that the subject could only be rigorously studied in a framework specifying the equations of motion of the system when not in equilibrium. He proposed as a formalization of Walras' idea of "tâtonnement" a set of price

adjustment equations that have formed the basis for nearly all later work.

I write those equations in an anachronistically general form as:

$$(2.1) \qquad \dot{p}_i = H^i(Z_i(p)) \quad \text{unless } p_i = 0 \text{ and } Z_i(p) < 0,$$

$$\text{in which case } \dot{p}_i = 0.$$

Here, p_i is the price of the ith commodity, Z_i is the total excess demand for that commodity, taken as a continuous function of the prices, and H^i is a continuous, sign-preserving function, bounded away from zero except as Z_i goes to zero. The dot denotes a time derivative.

In other words, the price of the ith commodity adjusts in the same direction as excess demand for that commodity, the exact adjustment being a continuous function of excess demand (and therefore of price). The exception to this occurs where such a rule would require that a good which is already free and in excess supply be given a negative price, whereupon the price is simply left at zero.

This formulation, which forms at least part of practically all later work, already raises some technical and some economically substantive problems. The first of these is easy to overlook. It is the assumption that excess demands can be taken as continuous functions of the prices. (Of course, this is to be interpreted as a continuous function of the prices given endowments, technologies, and so forth.) The problem is not that there are other variables held constant; it is that there are plausible circumstances in which excess demands <u>cannot</u> be taken as functions of the prices, let alone

continuous ones. If there are constant returns to scale, for example, a continuous increase in the price of the product, other prices constant, will cause firms to go from wishing to produce zero to wishing to produce an indeterminate amount to wishing to produce an infinite amount. Such cases are ruled out. One need not be a fanatic believer in the universality of constant returns to scale to be troubled by a formulation which cannot handle constant returns. Yet the assumption that excess demands are continuous functions of price is essential if we are to use the kind of mathematics which is embodied in (2.1), not only because it is hard to see what kind of adjustment process is reasonable in the absence of such a functional relationship but also because the very existence of solutions to (2.1) depends on such continuity.[1]

Thus we encounter important problems even in the building blocks from which the price-adjustment equation is constructed. When we come to that equation itself, we encounter another problem of even greater importance. Whose behavior does the equation represent? It seems very plausible, to be sure, that price should adjust upwards when demand exceeds supply and downwards in the opposite case, but just how does this happen? To put it sharply, in a world in which all participants take prices as given, who changes the price?[2] Indeed, in the center of a subject which deals with

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[1]Fortunately, a somewhat related technical problem involving the continuity of the adjustment process has recently been successfully handled. Claude Henry [19] and [20] has shown that the violation of Lipschitz Conditions involved in restricting prices to the positive orthant will still (under reasonable conditions) allow the existence of a continuous solution to (2.1), a matter which had previously to be taken on faith.

[2]This important issue was raised by Koopmans [22], among others.

individual behavior, how does there arise a behavior equation (not an identity) based solely on aggregates? The familiar story which goes with (2.1) of the auctioneer who adjusts prices until demand equals supply is at best an inconvenient fiction.

I shall return to these problems below. For the present, I add three remarks:

First, the difficulty arises directly with the price-adjustment equation used. It has nothing to do with the question of whether or not trade, consumption, or production takes place out of equilibrium.

Second, the difficulty arises because we have an excellent theory of equilibrium behavior and know very little about how individuals do or ought to behave when equilibrium is not present; hence, the resort to an aggregate equation.

Finally, despite this fact, we have already made an implicit assumption about disequilibrium behavior in the formulation of (2.1) that will have to be considered explicitly later on. It is one which is relatively harmless in the unrealistic world of no trading out of equilibrium but very irksome otherwise. Implicit in the assumption that excess demand influences price is the assumption that individuals take action to make their excess demands effective. This involves the assumption that they <u>can</u> take such action which, as we shall later see, implies that they have something of value which they can and do sell so as to have something to offer when they buy. It also involves, however, the assumption that they <u>do</u> take such actions and take them now, so that, even though the excess demand involved may be for a good dated in the far future, 1990 toothpaste, for example, individuals who will want that good in the far future begin <u>immediately</u> to attempt to acquire it.[1]

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[1]Or that individuals who will want to sell that good in the future begin immediately to attempt to sell it. The problems involved are particularly nasty as to firms, as I shall later discuss.

I shall refer to this as the "Present Action Postulate." It is implicit in all stability analyses, although explicit in almost none, possibly because it has little objectionable content so long as we remain in a world in which the adjustment of prices to equilibrium can be safely supposed to take place before the dates on any commodities (future or otherwise) come due.

Note that, even if we were to adopt a more individualistic and plausible price-adjustment equation than (2.1), the problems involved in the Present Action Postulate would still present themselves. Stability analyses which take off from the economics of individual behavior by way of deriving unsatisfied or excess demands must concern themselves with the question of when and how the participants attempt to exercise those demands. It is hard to see how to escape from the assumption that they do so when the demands occur, even though this involves implausible behavior as regards future commodities. I shall return to this below.

As indicated, however, this last set of problems can be ducked if one is willing to separate the time in which adjustment takes place from that in which commodities are dated and consumption and production occurs. This is essentially guaranteed if we suppose that no trading (let alone consumption and production) takes place before equilibrium is reached, so let us now return to a consideration of the historical development in which that assumption played an important role. It is, in fact, that assumption and not just the price-adjustment process (2.1) that characterizes the processes known as tâtonnement in the literature.

The early literature, beginning with Samuelson,[1] was primarily concerned

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[1] [29] and [30] and including especially Metzler [25].

with the local stability of the tâtonnement process, following Samuelson's observation that local stability properties could be studied by approximating the right-hand side of (2.1) linearly and considering the question of when the eigenvalues of the resulting matrix (essentially the Jacobian matrix of the excess demand functions) could be shown to have negative real parts. These discussions made little or no use of Walras' Law, but rather considered the excess demand functions as if they were pretty well unrestricted. Partly as a result of this and partly because of what now seems the rather peculiar origin of the subject -- the relations between what Hicks [21] had called "perfect stability" and the local stability of the truly dynamic (2.1) -- these investigations did not really get very far. They ended up in essentially the rather unfinished place of showing that, provided all commodities were gross substitutes,[1] the local stability of (2.1) was equivalent to the not very revealing condition that Hicks had found to be necessary and sufficient for his "perfect stability", namely, that the principal minors of the Jacobian of the excess demand function alternate in sign.

Such a result was obviously only a very partial one. This was so even in the narrow terms of the relations between Hicks' concept and dynamic stability where it remained for a much later paper of McFadden [24] to elucidate the matter in terms of relative speeds of adjustment. Even more partial was the result in terms of stability analysis on its own grounds, since the alternation of the principal minors is not particularly interpretable in any natural way. Such a result is best thought of as a lemma

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[1] That is, that a rise in the price of any one good increases the demand for every other, including income effects. For an analysis of what utility functions generate individual demand functions with this property, see Fisher [10].

rather than as a theorem, and, accordingly, we should not be surprised to find that by the end of the 1950's Hahn [15] and Negishi [26] had independently proved, using Walras' Law and the homogeneity of degree zero of the excess demand functions, respectively, that the assumption of gross substitutes itself implied the alternation of the principal minors and hence that, under the gross substitutes assumption, the tâtonnement process was invariably locally stable.

3. Tâtonnement: Global Stability

These papers, however, were superseded by the results of a pair of pathbreaking papers by Arrow and Hurwicz [4] and Arrow, Block, and Hurwicz [3] which contained a demonstration of the fundamental point that global rather than local stability results might be available (and, indeed, might be easier to obtain than local ones). Among the specific stability results was that of the global stability of tâtonnement under the gross substitutes assumption.

This breakthrough rested on the use of a mathematical device known as Lyapounov's Second Method. Since practically all the work that followed has been based on that device and since my later discussion requires some understanding of it, I shall digress at this point, give some not-very-rigorous definitions, and discuss Lyapounov's Second Method and the role it plays in analyses of stability.[1]

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[1] A more rigorous treatment can be found in Arrow and Hahn [2]. Lyapounov's original paper is [23]. The curious may wish to know that his first method (for proving stability) was to solve the differential equations being studied, an alternative that is never available at the level of generality involved in general equilibrium analysis.

An equilibrium of an adjustment process such as (2.1) is said to be globally stable if the system converges to it from every set of initial conditions. This, of course, is a very strong property. In particular, if there is more than one equilibrium, then no particular equilibrium can be globally stable in this sense, since if the process begins at some other equilibrium it will never leave it. Since we know that uniqueness of general equilibrium requires extremely strong assumptions, it is unreasonable to require equilibria to be globally stable. Instead, we need a somewhat weaker notion of global stability.

That notion is provided by the idea of the global stability of an adjustment process. An adjustment process, such as (2.1), is said to be globally stable if, for any set of initial conditions there is an equilibrium to which the system converges. The difference, of course, is that it doesn't have to be the same equilibrium for all initial conditions. If equilibrium is unique (as it is in the gross substitutes case), then the two notions of global stability coincide.

Finally, it is useful to distinguish this from the definition of the quasi-stability of an adjustment process.[1] For convenience of language, let us suppose that the variables involved are prices only, as in (2.1). Starting with any initial condition, consider an infinite sequence of prices. Such a sequence may not have a limit (unless prices converge to some equilibrium), but it may nevertheless have one or more limit points (roughly, points to smaller and smaller neighborhoods of which the sequence keeps returning). If every limit point of every such sequence is an equilibrium, the adjustment process is said to be quasi-stable.

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[1] So named by Uzawa [34].

Obviously, the quasi-stability and the global stability of an adjustment process are closely related. Indeed, it is not hard to show that if an adjustment process is quasi-stable and the variables involved remain in a compact set, then that process is also globally stable provided either of two things are true. They are, first, that every equilibrium is locally isolated (which includes uniqueness as a special case), or, second, that every limit point of every sequence starting from a given set of initial conditions is the same.

Accordingly, modern stability proofs tend to be in three parts. First (and usually hardest) is a proof of quasi-stability. (This is where Lyapounov's Second Method comes in.) Second is a compactness argument which often amounts to an argument that the variables remain bounded. Third is either a demonstration of local uniqueness of equilibrium or a demonstration that all limit points (starting from the same initial conditions) are the same.[1]

I shall have some occasion to mention the latter two steps in my later discussion, but our principal focus will be on the first one, the proof of quasi-stability and the use of Lyapounov's Second Method.

Continue to think in terms of prices. Lyapounov's result is as follows. Suppose that we can find a function, V(p), continuous in the prices, bounded below, and monotonically decreasing through time except in equilibrium. Then the adjustment process is quasi-stable. Lyapounov's Second Method consists

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[1]The work of Debreu [6] has shown that, for appropriate differentiability assumptions, local isolation of equilibria is true almost everywhere in the appropriate space of economies. Hence the last step in the stability argument might be made in this way. In practice, however, more direct results have generally been available.

of finding such functions $V(p)$. This is not a simple task and forms the heart of most stability investigations. As time has gone on, such Lyapounov Functions have tended to go from geometrically interpretable measures of the distance from equilibrium to economically more interesting functions such as (as we shall see) the sum of the utilities which households would expect to get if their mutually inconsistent plans could be realized.

Let us now return to the tâtonnement literature. Arrow, Hurwicz, and Block were essentially able to prove that the adjustment process (2.1) is globally stable provided that one is willing to place very severe restrictions on the excess demand functions. One such restriction -- that of the gross substitutes assumption -- has already been mentioned. Somewhat surprisingly, it turns out that this, as well as most of the other cases successfully investigated in the papers under discussion, are special cases of the assumption that the <u>aggregate</u> excess demand functions satisfy the Weak Axiom of Revealed Preference (at least in comparisons between equilibrium and other points), an axiom which makes excellent sense in terms of individual behavior but which there is no good reason to impose on aggregate demand functions.[1]

Despite this, because all the special cases involved turned out to imply global stability, the authors ventured to conjecture that the tâtonnement process was always globally stable, given only the restriction that the excess demand functions be consistent with underlying economic considerations

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[1]This can be said to be because of income effects. It is worth remarking that it is easy to see that at least local stability would be guaranteed if there were no income effects in view of the negative semi-definiteness of the substitution matrix. See Arrow and Hahn [2] for a more extended discussion.

such as Walras' Law. Not surprisingly, this rather wishful conjecture was soon exploded when Scarf [31] provided a counterexample.[1]

Tâtonnement stability was in an awkward corner. The process was not invariably stable; interesting necessary conditions for stability were clearly not to be had; and sufficient conditions for stability appeared to involve wildly restrictive conditions on individual (or aggregate) demand functions. Despite the fact that two other major developments were taking place at about the same time (the early 1960's) and would lead to what are (in my opinion) far more fruitful results, it is not surprising that the flurry of interest in the stability of general equilibrium more or less died out at about this time.

4. Trading Processes: Pure Exchange

Both of the new developments had in common the fact that, unlike the work just discussed which can be thought of as largely drawing implications for stability by considering properties which are just as true in equilibrium as out of it, they were based on further specification of the disequilibrium adjustment processes themselves.

One such development was the abandonment of the tâtonnement assumption of no trading out of equilibrium which had wound up in a dead end and the introduction of what are rather inelegantly called "non-tâtonnement" processes. Since that is not a very informative name, I shall refer to them as "trading processes."

[1] More recently, Sonnenschein [32] and [33] and Debreu [7] have shown that the underlying economic derivation of the aggregate excess demand functions does not imply any restrictions beyond homogeneity of degree zero and Walras' Law, thus eliminating any last forlorn hope that stability for tâtonnement processes might be aided by general rather than very special restrictions on the excess demand functions.

It is important to remember that, while this stage of development permits trading out of equilibrium, most of the work to be discussed does not permit consumption or production to take place until equilibrium has been reached. One must think of participants as swapping titles to commodity stocks while prices (and, of course, possessions) adjust. Only after the music stops do people go home and enjoy what they have. Such a model is obviously most suited to pure exchange with no firms. While a step in the right direction, it clearly calls out for extension both to firms and to a model which permits production and consumption to take place during adjustment.

For the moment, however, let us put such extensions aside and restrict our attention to the case of pure trading processes and pure exchange. At this level, a rather surprising result occurs. Since we have added to the price adjustment process (2.1) some set of equations describing how individual stocks of commodities change hands, we might expect that we have made stability less likely than in the tâtonnement case, since the system is now more complicated than before. This turns out not to be true, however, provided we stay with pure exchange and are willing to make the reasonable assumption that nobody can change his wealth by trading at constant prices. That latter assumption might be called one of "no cheating," for it amounts to saying that one cannot get anything by trading except by giving up something of equal value.

If one is willing to make that assumption (which I shall henceforth do), then it turns out that in essentially every case in which tâtonnement is known to be stable, exchange trading processes are also. The basic reason for this lies in Lyapounov's Second Method. Suppose that we have a

Lyapounov function which depends on prices and on aggregate excess demands. The time derivative of that function in a trading process differs from the corresponding time derivative in tâtonnement by terms which depend on those parts of the time derivatives of the aggregate excess demand functions which come from the changes in individual commodity stocks induced by trading with prices fixed. An individual's demand for any commodity, however, is affected by constant-price changes in his holdings only insofar as these changes affect his wealth, and, by the No-Cheating Assumption, his wealth is not affected. The change in an individual's excess demand, for a commodity brought about in this way is therefore merely the negative of the change in his actual holdings of that commodity. Since, in pure exchange, the aggregate stock of any commodity is fixed, there are no such effects on aggregate excess demand. Hence the time derivative of any such Lyapounov function is the same in tâtonnement as in exchange trading processes, so that if such a function was decreasing out of equilibrium in the former case, it is still doing so in the latter.

Unfortunately, this rather pleasant and general result does not take us very far. The difficulty with the tâtonnement results which might be said partially to have prompted the study of trading processes in the first place is that tâtonnement can be shown to be stable only under very restrictive conditions. So it is not a great help to know that those same restrictions will generally ensure the stability of general exchange trading processes too. Something more is needed.

That something more is provided by the fourth of the major developments which, in my opinion, have marked the history of the subject, the perception that relatively reasonable restrictions on the adjustment processes themselves

could lead to stability proofs involving essentially no restrictions on utility or production functions. While, as we shall see, that insight was closely and naturally associated with the development of the trading process literature, it is logically separate from that development. Credit for it (although they never enunciated it quite this way) belongs to Uzawa and to Hahn. (Indeed, one might say that it was Hahn who saw it most clearly, since he gave a tâtonnement example [16] in which the specification of (2.1) was restricted and quasi-stability proved under quite general circumstances.)

The result of this insight was the development of two special trading processes, the Edgeworth Process (so named because it was discovered by Uzawa [35]) and the Hahn Process (so named by Negishi [27]). I happen to believe that it is the Hahn Process which is the more interesting and fruitful by far, and most of the remainder of this talk will be based on it. Let me then take a little time to discuss the Edgeworth Process, which is also of importance.

The central assumption of the Edgeworth Process[1] is that trade will take place if and only if there is some group of people who can all be made better off by trading among themselves at the current prices. With some complications, due to the fact that at some non-equilibrium prices no such Pareto-superior move may be possible until prices change, this leads to the use of the negative of the sum of actually achieved individual utilities as a Lyapounov function and hence to quasi-stability.

Unfortunately, this powerful and appealing result is not so helpful as it first appears. This is so for more than one reason.

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[1]The principal works are Uzawa [34] and Hahn [17]. See also Arrow and Hahn [2].

First, while it is obviously innocuous to assume that individuals will not trade unless they can better themselves by so doing, it is not nearly so simple to assume that trade actually will take place whenever such a situation arises. This is because of the possibility that the only coalitions which can better themselves by mutual trade consist of very large numbers of people. Thus it is possible that there is no mutually advantageous bilateral or trilateral or quadrilateral trade and that the only mutually advantageous trade involves a very complicated swapping of commodities among millions of people. To require, as the Edgeworth Process does, that such a trade must take place is to put very heavy requirements on the dissemination of information and to assume away the costs of coalition formation.[1]

Second, the Edgeworth Process does not appear to lend itself readily to the important extension of allowing production and consumption to take place out of equilibrium. A crucial feature of disequilibrium consumption, for example, is surely that individuals take irreversible actions which they would not have taken had they correctly anticipated the future. This means that utilities turn out to be lower than anticipated. It is not easy to see how this phenomenon can be accommodated in a model whose central result relies on the increase of utilities out of equilibrium, although whether that can actually be done remains an open question.

It happens that the Hahn Process is strong on exactly these points. It requires much less information than does the Edgeworth Process; and it

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[1]The parallel to the theory of the core is obvious. David Schmeidler has shown (in an unpublished communication) that if there is any mutually advantageous trade there is one involving at most the same number of participants as there are commodities. While interesting, such a result does not really get out of the difficulty, especially when commodities at different dates are counted as different commodities.

is particularly suited to accommodating irreversible mistakes. In addition, it lends itself easily to the introduction of firms and its central assumption is almost compelled when we start considering individuals as adjusting prices. Accordingly, I now turn to an examination of the Hahn Process.

5. The Hahn Process

The central assumption of the Hahn Process[1] is as follows: At any one time, there may of course be either unsatisfied demand or unsatisfied supply for some commodity, say apples. However, we suppose that markets are sufficiently well organized that there are not both. In other words, there may be people who wish to sell apples at the current prices and cannot or there may be people who wish to buy apples at the current prices and cannot, but there are not simultaneously both unsatisfied sellers and unsatisfied buyers. Markets are sufficiently well organized that willing buyers and willing sellers can and do come together and consummate a trade very quickly, relative to the rate at which the disequilibrium adjustment equations operate (in the idealized abstraction, such deals are to be thought of as consummated instantaneously or outside of time). This requirement, while severe, seems to be a moderately reasonable one on information flows in a competitive economy; it is much less severe than the corresponding requirement in the Edgeworth Process.

Given this assumption, it follows that any individual who has a non-zero excess demand for a particular commodity finds that the aggregate excess

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[1]The original published paper is Hahn and Negishi [18].

demand for that commodity is of the same sign. Since prices adjust in the direction of aggregate excess demand, he finds that, outside of equilibrium, the things which he wishes to buy and cannot buy are getting more expensive, while the things which he wishes to sell and cannot sell are getting cheaper unless they are already free. Accordingly, he finds himself getting worse and worse off in the sense that his target utility, the utility which he expects to get if he can complete all his transactions, is going down. Accordingly, the sum of such target utilities will serve as a Lyapounov Function and quasi-stability can be proved.

Note the different roles played by utilities in the Edgeworth and Hahn Processes. In the Edgeworth Process the utilities actually achieved by individuals (that is, the utilities which they would attain if they now stopped trading) rise while trading is going on. In the Hahn Process, target utilities -- the utilities which they would achieve if they could complete their transactions -- fall out of equilibrium. One way of looking at it is to say that in disequilibrium plans are not all compatible and that, in the Hahn Process, the adjustment is such that people have to lower their expectations until equilibrium is reached and everyone can in fact attain the utility which he targets for.

One can go on from this to prove the global stability of the Hahn Process in pure exchange by establishing or assuming the boundedness of prices and then showing that all limit points of any sequence starting from a given set of initial conditions are the same. The latter point is proved rather nicely by using strict quasi-concavity of the indifference maps,[1] but I shall not linger over the details now.

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[1] Arrow and Hahn [2] were the first to do this in their somewhat more satisfactory version discussed below.

So far, so good. Unfortunately, there are problems with this simple version (beyond those common to all such processes, which I discuss below). Thus, suppose that there are at least three commodities, say apples, bananas and carrots (the need for at least three arises from Walras' Law). Suppose that, at current prices, apples and bananas are both in aggregate excess supply. There may be an individual who would like to buy bananas but has nothing to offer for them but apples. If he is one of the unlucky ones who cannot find a buyer for his apples, then, regardless of how easily he can find a willing banana seller, no trade will be consummated, for he can offer that seller nothing which the seller regards as having any value. Out of equilibrium, this can occur even if apples have a positive price. No amount of efficiency in market organization will get around this and the Hahn Process depends on it not occurring.

This possibility, which depends on the need to sell before you can buy, was first recognized by Clower [5] among economic theorists working in a somewhat different context. It is well known to historians, however, that economies which rely on the Hahn Process can run into this difficulty. As the ancient document has it:[1]

> "Simple Simon met a pieman going to the fair.
> Said Simple Simon to the pieman, 'Let me taste your ware.'
> Said the pieman to Simple Simon, 'Show me first your penny.'
> Said Simple Simon to the pieman, 'Indeed, I haven't any.' "

It was not enough for Simple Simon and the pieman to be able to find each other. Trade still did not take place.

You will note, however, that the Simple Simon economy already had in it one feature which distinguished it from the models I have been talking

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[1] M. Goose [14].

about, the use of one commodity as the sole medium of exchange. No doubt reflecting on this, Arrow and Hahn [2] were able not so much to solve as to isolate the Simple Simon problem by constructing a rather more satisfactory version of the Hahn Process model in which one commodity (which they happened to call "money") serves as the only such medium. In that version, "target" excess demands -- the excess demands which would obtain without a money constraint -- are distinguished from "active" excess demands -- the excess demands which are expressed by actual offers to buy or sell, offers to buy requiring money to back them up. It is aggregate active excess demands which then influence prices in (2.1). It is natural to take the Hahn Process assumption as applying to active demands since it then really does become merely one of market organization. Arrow and Hahn were able to show the global stability of the resulting Hahn Process, provided that nobody ever runs out of money and that every individual who has money actively tries to satisfy some non-zero fraction of every non-zero target excess demand that he has.

The No Bankruptcy Assumption is, of course, very strong; moreover, it appears indispensable and it is hard to know how to assure that it holds,[1] although, as we shall see, it becomes more palatable when we start to let individuals rather than auctioneers control prices. Moreover, the Arrow and Hahn treatment points up two more difficulties which are common to the entire literature but which now begin to surface more explicitly. While we have referred to them once, it is useful to mention them in the present context.

The first such difficulty concerns what I have called the Present Action Postulate. Households in this model must act now to make some non-zero part

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[1] For some discussion of this, see Arrow and Hahn [2]. See also Fisher [9] discussed below.

of every non-zero target excess demand an active excess demand. Money must now be allocated to an attempt to buy a non-zero amount of every commodity which the household does not have enough of, even though the household may already have a large amount of that commodity and relatively little of another one which it also wants to buy. Note that it would be harder to get away with this if the household were simultaneously consuming its existing commodity stocks.

The Present Action Postulate seems as innocuous as it does in this context, however, mainly because we have already implicitly swallowed the camel of assuming that the household believes its transactions will be consummated before consumption time comes and is therefore indifferent about the order in which it attempts to complete them. This is a part of assuming that the household really does not notice that it is out of equilibrium, that it formulates its demands on the assumption that prices will not change, and that it rather stupidly does not notice what is really going on. Such problems are the natural consequence of having a good equilibrium theory and not knowing much about individual reactions to disequilibrium.

Nevertheless, the Arrow-Hahn introduction of money as a sole medium of exchange is a considerable step forward. It clarifies and isolates the Simple Simon problem in the No Bankruptcy Assumption; it begins to bring forward the problems just discussed; it has a modicum of realism, if not of monetary theory; and it is probably indispensable for the introduction of firms into trading processes.

This last point is true for the following reason. Without the introduction of money, firms would value their holdings the same whether or not they sold them. A firm which produced a large stock of toothpaste, for

example, would feel it was doing well in terms of profits if toothpaste had a high price. There would be nothing (absent further assumptions) to make it interested in selling the toothpaste as opposed to holding it and therefore no reason why the presence of that stock of toothpaste should affect the price any more than the presence of target demands which are not active. Requiring firms to keep score in money which is needed by themselves and especially their owners to back up demands assures that firms actively participate in the market process.

As it happens, the Hahn Process Model can be fairly readily extended to accommodate firms (but not yet production). To this I now turn.

6. Firms (But No Production) in the Hahn Process[1]

The obvious first question that occurs when one seeks to introduce firms into a model which permits trading but no other economic activity to take place before equilibrium is reached is that of what it is that firms are to trade. In pure exchange, the stocks of commodities are fixed and one can think of households swapping titles to various amounts of them; with firms in the picture, however, one has to think of the transformation of inputs into outputs as at least envisioned by the participants in the economy, even if we do not allow that transformation to take place during the adjustment process.

The answer (which will continue to be helpful when production is introduced) is to think of firms as trading in commitments to buy inputs and sell outputs. Each firm has access to a production technology which describes

[1]This discussion is based on Fisher [12].

the efficient ways in which inputs can be turned into outputs. While trading is going on, the firm contracts for inputs and commits itself to deliver outputs. Payment for all such contracts and commitments is made when they are signed, but delivery and production do not take place until equilibrium is reached. The firm trades in such commitments at current prices, seeking to maximize profits while ending up in a technologically feasible position. Since, out of equilibrium, a firm may not be able to complete all the transactions which it attempts, it will sometimes be the case that the firm finds it has committed itself to sell outputs without in fact having acquired the inputs which would enable it to do so. In that case, it must go back on the market and either acquire the needed inputs or buy up output commitments, whichever is most profitable at the ruling (presumably changed) prices. Similarly, a firm which has overacquired input commitments will wish either to sell them off again or to sell further output commitments, whichever is most profitable.

Firms in this model pay out some or all (but not more than) their actually realized profits as dividends to their owners, although they may (and sometimes must) keep some retained earnings for transactions purposes so long as equilibrium has not been reached (the No Bankruptcy Assumption). Since everyone continues to believe that all transactions will be completed at current prices, however, households maximize their utilities and formulate their <u>target</u> excess demands on the basis not of received dividends but of the expected total earnings of the firms they own. Dividends only matter in providing some of the cash which backs up target demands and makes them active. Indeed, it is easy to see that Walras' Law only holds in terms of target demands and target profits and not in terms of active demands and actual profits.

It is also not hard to show that Walras' Law requires that all profits be paid out to owners in equilibrium, so that any apparent tension in these matters, important as it is, is a disequilibrium one.

Now, what about stability in such a Hahn Process model? A crucial feature of firms which differentiates them from households is that their maximand, profits, involves prices as well as quantities directly. Nevertheless, it remains true of the firm, as of the household, that everything which it wishes to buy but has not yet bought is getting more expensive while every non-free good which it would like to sell is getting cheaper. Accordingly, target profits are going down out of equilibrium.

Given this fact, one can go on to consider the target utility of the household. So far as its trading is concerned, it too is caught in the same Hahn Process trap of wanting to buy goods whose prices are going up and wanting to sell goods whose prices are going down. The only way it can be getting better off (in terms of target utility) is for the firm it owns to be becoming more profitable. We have just seen, however, that firms become less profitable rather than more and accordingly, it is still the case, as it was in pure exchange, that the target utilities of households are falling out of equilibrium. This means that one can once again take the sum of target utilities as a Lyapounov function and establish quasi-stability.

A full proof of the global stability of the adjustment process, however, requires two more steps, as remarked above: first, a proof of compactness;

and second, a proof either that all limit points are locally isolated or that, given the initial conditions, all limit points are the same. I have rather slighted these steps in my earlier discussion, and it now seems appropriate to spend a little time on them since the introduction of firms begins to make them rather more complicated.

First let us consider the issue of compactness. Since we have so far been working in Euclidean space, this is basically an issue of boundedness, for, if we can show that the variables remain in a bounded set, there is no harm in taking them to lie in its closure. The basic variables here are the prices and the stocks or commitments of the commodities held by individual households and individual firms.

The problems involved in assuring that prices remain bounded are basically unaffected by the introduction of firms and, indeed, are the same for trading processes as they are for tâtonnement. (This is largely because the same price adjustment process (2.1) is used in both cases.) Aside from the alternative of assuming directly that prices are bounded -- which is not all that unappealing -- there are two known ways of going about it. The first of these is to consider the classic case in which the price adjustment functions, $H^i(\cdot)$, which appear on the righthand side of (2.1) are linear through the origin. Whether we work in terms of absolute prices or in terms of prices relative to that of the numéraire (money in the Hahn Process model), it is not hard to show that (provided the numéraire would be in excess demand if it were priced at zero -- an innocuous assumption) there is a hyperellipsoid in the space of the prices, outside of which prices cannot go. If the units in which quantities are measured are appropriately chosen, this

amounts to showing that the sum of squares of the prices is bounded above. The same result can be extended to the case in which the price adjustment functions, $H^i(\cdot)$, are not necessarily linear themselves, but are bounded above by rays through the origin.

This is a strong result which basically depends on limiting speeds of adjustment in a particular way. Boundedness of relative prices can also be accomplished without such a limitation if one is willing to restrict demands. Thus, it is possible to show boundedness if one is willing to assume that whenever any subset of prices become high enough relative to all others, then the highest-priced good is in excess supply. Such an assumption attempts to capture the notion that it is possible to price oneself out of the market, but it is pretty strong.[1]

Boundedness of individual commodity or commitment holdings is a potentially more delicate matter than that of prices, although it is relatively simple in the contexts we have so far considered. Under pure exchange there is, of course, no problem since total commodity stocks do not change. Once we introduce firms, however, there is at least the large potential difficulty that firms may wish to undertake unbounded commitments, even though they cannot in fact do so. This would occur, for example, under constant returns since firms are not supposed to take resource limitations into account directly. As it happens, however, constant returns have already had to be ruled out of the models we have been discussing because of the necessity of having excess demands continuous functions of the prices and, indeed, in the context of the Hahn Process model with firms but no

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[1]The possibility of proving boundedness of prices this way was first noticed in Fisher [9] but the first correct version is in Fisher [12].

production, it is in any case necessary to assume that each firm has a unique profit-maximizing position continuous in the prices. Since prices are to be bounded, such an assumption already implies boundedness of commitments and gets us out of the problem.

It would be very shortsighted to let it go at that, however, both because an extension to constant returns is a clear direction for further work and because the introduction of consumption and production to be discussed below makes the problem one of bounding the integral of commitments rather than commitments themselves. The most attractive way to ensure boundedness of commitments (or their integrals) without the strong assumptions just mentioned seems to me to be to use what we know about the role of the price system in a world of limited resources. As resources become scarce, their prices ought to rise. Accordingly, it is undoubtedly possible to show that unbounded commitments cannot be profitable since the unit costs of production would rise above the price at which output can be sold. Such an argument would require both the boundedness of prices and a fairly careful specification of the technology, making limited or primary resources a non-negligibly required input for every good, either directly or indirectly. It also turns out to require care in restricting speeds of adjustment. Whoever works out the constant returns case is going to have to pay more detailed attention to this than I have, since I have hitherto been content with sketching out the way such an argument should go.

Returning now to stability in the Hahn Process model with firms (but no production), the final step is a proof that, given the initial conditions, all limit points are the same. In the case of pure exchange, Arrow and Hahn [2] were able to do this by using the strict quasiconcavity

of the utility function. I have adapted their proof to the present case. What is involved is essentially the use of the fact that, at equilibrium prices, households are minimizing expenditure for given utilities while firms are maximizing profits, given their technologies. However, since target profits and target utilities decline out of equilibrium, and since every limit point is an equilibrium (quasi-stability), each household has the same target utility and each firm the same target profits at any one limit point equilibrium as at any other limit point equilibrium. At the prices of equilibrium A, therefore, the commitments of equilibrium B would be worth less to firms than the commitments of equilibrium A itself. On the other hand, those same equilibrium B commitments must cost households more than would those of equilibrium A. Since the net commitments of the production sector are made only to the household sector, and vice versa, this establishes a contradiction in a rather pretty way unless the equilibria are the same as regards commitments. One can then go on and complete the proof by a fairly straightforward argument as to prices and marginal rates of substitution.

7. Consumption and Production Out of Equilibrium[1]

We now come to the obviously important step of allowing real economic activities other than trading to go on before equilibrium is reached. While, as I shall describe in a moment, I have produced a Hahn Process model in which disequilibrium consumption and production are allowed to take place, perhaps the most illuminating part of working on such a model turns out to

[1] This section is largely based on Fisher [13].

be not the resulting stability proof, but the way in which a number of underlying issues are pushed into the open when consumption and production are to be included.

Let me begin, however, by describing the way in which the model just referred to operates. Every household has a utility functional (it is convenient to work in continuous time) which is defined over its past and future consumption profiles of all commodities. At any given moment, however, the past is irreversible and the household can only attempt to optimize with respect to consumption yet to be undertaken. (In a sense, the utility function of the household is allowed to change as a result of its past consumption.) It performs such optimization assuming that current prices will last forever and subject to a budget constraint which involves its share of the target profits which it expects to receive from firms.

Firms are analogously situated. The technological opportunities open to a firm at any given time depend in part on the input and output actions it has taken in the past. Given its own past actions, the firm attempts to maximize profits with respect to its planned pattern of inputs and outputs, also assuming that prices will be constant.

The essential difference between this model and the preceding one with firms but no production is that firms and households take irreversible actions which change their optimizing actions later on. In the case of the household, such actions change the utility function; in the case of the firms, they change the set of feasible production points; in both cases, they change the stock of commodities available for trade. We know, however, that if such actions were reversible, then target profits for firms and target utilities for individuals would decline out of equilibrium, given

the other assumptions of the Hahn Process model, for we would then be back in the previous case. However, profits for firms or utilities for households can never be higher given the restrictions imposed by irreversible actions than they would be could those actions be undone at will. It is therefore not hard to show that target profits for firms and target utilities for households are still declining out of equilibrium; indeed, the declining utility and profit feature of the Hahn Process makes it particularly adaptable to the case of disequilibrium consumption and production.

Given this result, it is possible to go on to prove the global stability of the adjustment process along the lines of the previous model, although there are some technically rather tricky details. These are related to two properties of equilibrium in this model which are of substantive interest and which I shall thus briefly describe.

The first of these properties is that trade actually ceases at equilibrium. In other words, equilibrium does not involve a situation in which everyone finds that he is able to go on trading as he planned to do at prices which he expected to rule, but rather a situation in which all trades have already taken place and the only remaining activities are the carrying out of consumption plans by households and the meeting of already paid for commitments by firms.

Second, because the optimizing positions of households and firms change as a result of irreversible actions, the set of competitive equilibria at any moment depends not merely on current possessions and prices but also on past history. Hence the process converges to an equilibrium which is also moving.

As already stated, however, the interesting thing about this model may not be the results but the problems which it pushes to the fore. I have already discussed some of these in the context of earlier work, but it is now appropriate to consider them again.

The first of these problems has to do with the dating of commodities. It is common and sensible to suppose that the same commodity at two different dates is to be treated as two different commodities. On this view, dates on commodities matter.

The question arises, however, of whether or not the dates on commodities are reached during the adjustment process. So long as we remain in the unreal world of tâtonnement and even so long as we permit trading but no other activity out of equilibrium, we can suppose that those dates are not reached, although this is a bit sticky in the latter case. As soon as we permit production and consumption to take place during the adjustment process, however, we are forced to assume that some commodity dates are reached before equilibrium. One cannot now consume tomorrow's toothpaste.

The reason that this creates a difficulty lies in the fact that trading in past as opposed to future goods does not take place combined with the assumption that participants expect their trades to be consummated and fail to realize the disequilibrium nature of their circumstances. Hence, at the moment that a particular commodity date is passed, trading in that commodity is suspended and, out of equilibrium, there will be individuals who expect to trade further in that commodity and now suddenly realize that they cannot do so. Such individuals will now have to reoptimize subject to the suddenly imposed constraint that they are stuck with whatever they have of the dated commodity in question. Since, out of equilibrium,

that constraint will be unexpected and will generally matter, its imposition can easily cause a discontinuity in the excess demand functions. This violates Lipschitz Conditions and removes the mathematical foundation for most stability analysis.

Now we know perfectly well why this problem sounds somewhat contrived. It seems obviously unreasonable for someone to care at midnight on December 31 that he cannot buy last year's toothpaste when he can obviously buy a perfect substitute, namely, this year's toothpaste. A little reflection, however, reveals that this is not an adequate way out of the difficulty. The reason that I am generally indifferent between buying last year's and buying this year's toothpaste as of the transitional moment is that I generally find that they are trading for the same price as of that moment. There are, of course, excellent reasons why that should be so, but they are mostly equilibrium reasons. There is certainly nothing in the price adjustment mechanisms of the models so far discussed that guarantees that prices will come together by the time the transition takes place. It is not merely a matter of permitting arbitrage but also of building a model in which arbitrage is sufficiently effective to erase such discrepancies by the crucial times. No one has yet done this and it does not appear to be particularly easy.

One way around this difficulty may be to look more closely at the price adjustment process which we have been using. Suppose we abandon the fictional auctioneer and allow sellers to fix their own prices with buyers searching for the optimal price, a class of models which I discuss below. Then it is natural to identify commodities not merely by physical description and by date but also by the name of the dealer who sells them.

If we do that we run into other difficulties, but we may evade the dated commodity problem, for, with a single individual adjusting the price of today's and tomorrow's toothpaste and also seeking to trade in those goods, arbitrage will in fact be instantaneous. Such an individual will take care to see that those prices come together as today blends into tomorrow, for it will be to his advantage to do so.

I cannot say, however, that I have worked out such a model and, indeed, in the model described above, I assumed away the dated commodities problem by assuming that all transitional moments are smooth or, in the extreme case, that future and spot prices for the same commodity are identical so that there are no dated commodities. This is obviously unsatisfactory and it exacerbates another problem, that of the Present Action Postulate.

We have already seen that it is crucial in a model in which prices are driven by excess demands that such demands be expressed currently. Just as demands which are not backed up with purchasing power can have no effect on price, so too demands which remain merely gleams in the eye of the demander can have no effect. Yet all stability proofs assume that it is the excess demands derived from optimizing calculations that the demander attempts to exercise at least to some degree. We must now examine this more closely.

Let us begin with households. In all the models we have considered, participants believe that current prices will rule forever, since they are not aware of disequilibrium. This means that a household believes that it can always trade toothpaste at the current price. If it currently has a sufficient stock of toothpaste to satisfy its needs for some time to come but expects to need toothpaste sometime in the future, why should it <u>now</u>

begin to acquire the extra toothpaste rather than wait until it is needed? (I abstract from questions of perishability and storage costs.) This seems particularly strange if we allow the household sufficient consciousness of what is going on that it becomes aware that it has a cash constraint. Indeed, since the household expects to be able to trade at the same price forever, its purchase plans are not determined even though its consumption pattern is, since it can buy more toothpaste than it needs and sell off the excess later or do the reverse, acquiring cash in the process.

This problem is present whether or not commodities are dated but is worse if they are not than if they are. It is bad enough to require that a household which expects to need toothpaste in 1980 should immediately enter the futures market for 1980 toothpaste and attempt to acquire it; it seems even more unreasonable to require that such a household should attempt to make such acquisitions in a world without dated commodities or futures markets by immediately entering the spot market for toothpaste. The best that can be said for this is that it forces the household to take a particular one of a set of equally optimal actions, given the assumptions under which it operates. Some vague notions as to transactions costs can be imported from outside those assumptions to justify this.

Unfortunately, not even such handwaving will do for firms, if commodities are not dated. For firms, the Present Action Postulate requires that they look into the future and consider whether they will end up being a net supplier or purchaser of any given commodity. If they find that over all future time it is best for them to sell a particular commodity, then they have to begin to offer it for sale right away. This is unreasonable both because

their output of that commodity may not become positive for a long time to come and because it rules out technologies in which a given firm can find it optimal at given stationary prices first to buy a good for use as an input and then later to produce more of it as an output. Such problems are less when commodities are dated but, even then, the problems remain the same as for households.

Clearly, a principal source of such difficulties is the continuing assumption that participants fail to realize that they are in disequilibrium. We have no adequate theory of disequilibrium behavior and all these models impose equilibrium-derived behavior on a disequilibrium process. This is obviously unsatisfactory wherever it appears.

8. Individual Price Adjustment

For my final topic, I consider the question of dispensing with the fictional auctioneer who lurks behind the price adjustment process (2.1) and allowing individuals to set prices. This is, I believe, a very deep matter, closely related to the problem of how to allow consciousness of disequilibrium to occur in the behavioral assumptions of stability models. For, as I have already remarked, in a world in which everyone assumes that prices are given, how do prices ever change? Indeed, it is a very serious question whether or not one can have individualistic price setting models together with consciousness of disequilibrium and still end up at the competitive equilibrium. This is as much of a problem for analyses of single markets as it is for general equilibrium analysis, although we perhaps know more about it in the partial context.

The obvious thing to do, if one is going to have individual price adjustment, is to assume that participants on one side of each market set their own prices and then that participants on the other side search over prices to find the most advantageous one.[1] For convenience, I shall speak of the price-setters as the sellers and of the searchers as the buyers, but there is no reason why it can't be the other way round in particular markets.

If one considers such a model in the general equilibrium context in which we have been working,[2] it becomes natural to deal with the wares of different sellers as different commodities, even if they are physically identical. This, of course, raises the issue of whether or not one can ensure that the prices of two such "seller-differing" commodities (i.e., the prices charged for the same commodity by two different sellers) will be the same in equilibrium. Not surprisingly, they will be if buyers are good at searching and they won't be if some sellers have persistent locational advantages in terms of buyer search, in which case the difference between the commodities is in some sense real even in equilibrium.

The interesting thing about distinguishing commodities in this way, however, is that, if one does so, the Hahn Process assumption becomes so natural as almost to be compelled. In such a context, that assumption amounts to saying that, at any one time, if a given seller (who is the only one on the supply side of the market for the commodity he sells) finds that

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[1]There is a burgeoning literature on such search models, most of which are not directly concerned with the question of whether one can end up in competitive equilibrium but within the no less interesting question of where one does in fact end up. For a good critical survey, see Rothschild [58].

[2]The ensuing discussion of the general equilibrium issues is drawn from Fisher [9].

he cannot sell as much as he would like to sell at the price he sets, then there are not also buyers (with money) who know about him and would like to buy more from him at that price. Similarly, if there are such buyers, then the seller can sell all he wants. This is so clearly reasonable as to provide an independent very strong reason for interest in the Hahn Process.

There is another reason as well which stems from assuming individual price adjustment. Perhaps the most bothersome assumption in the Hahn Process model is that of No Bankruptcy, that participants do not run out of money before equilibrium is reached. Now, when prices are controlled anonymously, there are two ways that individuals can run out of money. First, they may have spent all their money and have for sale only things now valued at zero prices; second, they may have spent all their money and have for sale things which have positive prices but which are in oversupply. There is nothing about individual price adjustment which takes care of the case in which someone is offering a zero-priced good, but the other case, of goods with positive prices but in oversupply can be handled. The reason for this is that such cases are essentially ones in which a seller is pricing himself out of the market. When prices are anonymously controlled, the seller can do nothing about this, but when he has control over his own price he can lower it and can lower it the more quickly the closer he is to bankruptcy. Hence the No Bankruptcy assumption, while still strong, becomes more palatable when we allow individual price setting to take place.

Thus the Hahn Process, for two reasons, becomes an appealing one in an individualistic price adjustment context. Indeed, I have shown [9] that (with some complications involving the switching of buyers among sellers

as search proceeds) the Arrow-Hahn version of the Hahn Process in pure exchange can be extended to allow individual price adjustment while still proving stability. There seems little doubt that the same can be done for the more complicated versions involving firms, consumption, and production which I have already discussed.

There is one big catch to this, however, and that is the issue of just how individuals set prices in disequilibrium. The extension of the Hahn Process results to individual price adjustment just mentioned depends on sellers being rather foolish about what is happening. It requires each seller to set a price believing that there is a market equilibrium price and then to adjust that price up or down depending on whether he finds he can sell more or less than he wished at that price. In other words, the seller behaves like a little auctioneer and adjusts his own price according to the excess demand which occurs on his own personalized market. While one can rationalize such behavior in terms of a rule of thumb for finding an unknown equilibrium price, this really is not very plausible. As Rothschild [28] has pointed out, not only is it unreasonable to make sellers act while setting such prices just as they would if they were certain that they had set equilibrium prices but also it is hard to suppose that sellers fail to notice that their demand curves are not flat and that the number of searching buyers who attempt to purchase from them depends on the prices they set.

These difficulties, of course, are but part of the general one to which I have already referred, namely, that all these models make the participants base their behavior on the obviously erroneous belief that equilibrium has been reached and their transactions will be consummated. We simply lack a satisfactory theory of individual behavior in disequilibrium which is

powerful enough to give general equilibrium stability results. Nevertheless, the problem seems particularly acute as regards price adjustment behavior, perhaps just because we do have some notion as to how price setters ought to behave in such a situation. Putting aside the uncertainty issue (which is very difficult) it appears evident that sellers facing a declining demand curve, as these do, ought to behave as monopolies.

What happens if they do? Is it nevertheless possible to get convergence to competitive equilibrium or does some residual monopolistic element remain in any search model?[1] So far as general equilibrium is concerned, this is an important but wholly open question. The problem, however, arises also in the context of a single market and here one seems able to make a little progress.[2]

Let me thus restate the problem in the context of a single market so as to emphasize how fundamental it is. Just about the first thing any beginning student of economics encounters is a demonstration that competitive markets end up where supply equals demand. He is told that, if price is such that supply exceeds demand, then there will be unsold goods and sellers will offer lower prices to get rid of them; similarly, if demand exceeds supply, unsatisfied buyers will bid up the price. This basic idea lies behind the price-adjustment process (2.1) which we have been discussing. Like that process, however, it presupposes a model of how price gets changed out of equilibrium and it is hard to come to grips with this in a context in which everyone takes price as given. Even at this level, we lack a satisfactory fully rigorous disequilibrium model of price adjustment.

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[1]As suggested by Arrow [1].

[2]Indeed, practically all the search literature is about single markets. The ensuing discussion is based on Fisher [8] and especially [11].

Suppose then that we try to build such a model by allowing sellers to set prices and buyers to search for low prices. If sellers behave in the not very sensible way indicated above for general equilibrium, setting prices, behaving as though their demand curves were flat, and adjusting prices according to whether or not they sell out all their supplies, then it is not hard to show, given reasonable restrictions on the search behavior of buyers that the process can be made to converge to competitive equilibrium.[1]

Suppose, however, that we allow sellers to realize what is happening and to know the declining demand curves they face. Can we nevertheless get to competitive equilibrium allowing sellers to use that information in a profit maximizing way? It turns out to be possible to tell such a story with a competitive ending, but the only such story which has been successfully told, so far as I know, turns out to be very complex and perhaps something of a fairy tale. It depends on demand curves flattening out at low enough relative prices as buyers find firms with unusually low enough prices and on buyers gradually learning where the minimum price they can find can be expected to be if it persists long enough.[2] Whether even such unsatisfactory results can be carried over into the analysis of general equilibrium stability is an open question.

I end then where I began. The analysis of the stability of general (and even partial) equilibrium is in a far from satisfactory state. Although considerable progress has been made, such results as we have depend crucially on assuming that participants behave as though they were in equilibrium and do not realize what is going on. We lack a theory of convergence to

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[1]See Fisher [8].

[2]See Fisher [11] .

equilibrium based on more reasonable individual behavior largely because we usually do not know what more reasonable individual behavior is. It is questionable whether a satisfactory model of individual disequilibrium behavior will in fact converge to competitive equilibrium.

Nevertheless, the issue is one of great importance to economists. We all continually behave as though the subject had long ago satisfactorily converged.

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BASEMENT

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